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MA-101/1841

B. Tech. (Semester-I) Examination-2014

Mathematics- I

Time: Three Hours] *[Maximum Marks:* **100**

Note: Attempt questions from all the sections.

Section-A

(Short Answer Type Questions)

Note: Attempt any ten questions. Each question carries 4 marks **(4x10=40)**

1. Show that $A = \begin{bmatrix} -i & 3+2i & -2-i \\ -3+2i & 0 & 3-4i \\ 2-i & -3-4i & -2i \end{bmatrix}$ is skew Hermition matrix

2. Find the rank of matrix

$$\begin{bmatrix} 2 & 4 & 3 & -2 \\ -3 & -2 & 1 & 4 \\ 6 & -1 & & 2 \end{bmatrix}$$

3. Using matrix method show that the equations $3x + 3y + 3z$ [illegible] $= 4$ $10y + 6z =$ [illegible] & $2x$ $3y$ are consistent

4. If $y = a\,Cos(logx) + b\,Sin\,(logx)$ then show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n \approx 0$

5. If $\mu = e^{xyz}$ find the value of $\frac{\partial^3 \mu}{\partial x\,\partial y\,\partial z}$

6. Trace the curve $y^2(2a - x) = x^3$.

7. In estimating the number of bricks in a pile which is measured to be $(5mx\ 10m\ x5m)$ count of bricks is taken as 100 bricks per m^3. Find the error in the cost when the tope is stretched 2% beyond into standard length the cost of bricks is Rs 2000 per thousand bricks.

8. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

9. Calculate the volume of the solid bounded by surface $x = 0, y = 0, x + y + z = 1$ & $z = 0$

10. Show that $\int_0^{\pi/2} (\sqrt{\cot\theta})\,d\theta = \frac{1}{2}\sqrt{1/4}\sqrt{3/4}$

11. Using Green's theorem evaluate $\int_c (x^2y\,dx + x^2dy)$ where c is the boundary described counter clock wise of the triangle with vertices (0,0), (1,0), (1,1).

12. Evaluate $\iint_S (yzi + zxj + xyk)ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.

13. Evaluate $\int_0^a \int_{y^2/a}^{y} \frac{y}{(a-x)\sqrt{ax-y^2}} dx\, dy$ by changing the order of integration.

14. Find the divergence of the vector field $\vec{V} = (x^2 - y^2)i + 2xyj + (y^2 - xy)k$

15. Use Lagrange's method of undetermined multipliers to find the minimum value of $x^2 + y^2 + z^2$ subject to the conditions $x + y + z = 1; xyz + 1 = 0$

Section-B

(Long Answer Type Questions)

Note: Attempt any three questions. Each question carries 20 marks. (20x3=60)

1. Find the characteristics equation verify Cayley Hamilton theorem & hence find A^{-1} of matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

2. Expand $x^2y + 3y - 2$ in powers of $(x-1)(y+2)$ using Taylor's theorem.

3. If $y_1 = \frac{x_2x_3}{x_1}$, $y_2 = \frac{x_3x_1}{x_2}$, $y_3 = \frac{x_1x_2}{x_3}$

show that the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 is 4.

4. Evaluate $\int_0^{\log x} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\, dz\, dy\, dx$

5. State the Gauss divergence theorem, verify the theorem for $\vec{F} = (x^3 - yz)i + (y^3 - zx)j + (z^3 - xy)\hat{k}$ taken over the cube found by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

6. Find the eigen values & eigen vectors of the matrix:

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$